# Parametric study of an elliptical fuselage made of a sandwich composite structure

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**Abstract**

We size the shell thickness of a pressurised elliptical fuselage and analyse the weight gains or savings compared to a circular fuselage. Three fuselage construction cases are analysed: a monolithic construction, a symmetric sandwich with two facesheets of equal thicknesses separated by a lightweight core, and an unsymmetric sandwich with two facesheets of different thicknesses. We develop a proper dimensionless analysis comparing the proposed elliptical fuselage with a circular one for two different scenarios: equivalent cross-section areas and enclosing a similar rectangular box. We apply a semi-analytical thin shell theory to compute the loading due to pressurisation around the circumference of the fuselage cross-section. We select the facesheet thicknesses above and below the sandwich core at every location to minimise weight. The goal is to compute the structural weight gain or penalty that is incurred by replacing a circular fuselage with an elliptical one to resist internal pressurisation in function of the scenario, eccentricity, fuselage diameter, and sandwich core thickness. We find that an elliptical cross-section incurs a significant penalty in terms of necessary facesheet thickness even with an optimised unsymmetric sandwich construction. This penalty can be minimised by keeping the eccentricity low, the loading intensity low and the core thick.

*Keywords:* elliptical fuselage, non-circular fuselage, pressure vessel, sandwich structure

## 1. Introduction

It is now well known that the use of composite materials in aircraft structures leads to weight and cost savings. Yet, the aerospace industry does not take full advantage of these materials as composite structures are often manufactured based on their metallic counterpart designs. The composite plies are stacked up to make symmetric and balanced lay-ups in order to avoid coupling between bending and twisting loadings and in-plane deformations. In addition, most composite structures look very much like the metallic structures that were seen in the past. This is the case for pressurised fuselages. Although the manufacturing processes are different for composites, the structure obtained looks the same as a metallic structure: a thin skin reinforced by frames and stringers [1].

To use composites in a thin skin construction makes sense for structures subjected to in-plane loadings, like a circular pressurised fuselage. In effect, in a circular fuselage, the internal pressure is balanced by purely tensile loading of the membrane. Although this circular cross-section geometry is ideal as a pressurised vessel, aircraft designers must consider other factors in the selection of a fuselage geometry: maximised interior space for passengers comfort, below-floor space for system wiring, aerodynamic considerations, etc. [2, 3, 4, 5, 6]. These criteria may lead to the selection of an oval shape as a fuselage cross-section or even a blended wing body (BWB), with the associated generation of bending moments upon pressurisation. The fuselage construction must then be adapted to effectively resist these bending moments.

Composite materials, used in a sandwich construction, may be an effective solution to this problem. In a composite sandwich construction, the stiff carbon fibre facesheets are separated by a lightweight core, increasing the moment of inertia of the cross-section and thus, the bending stiffness of the structure. Rectangular fuselage cross-sections with rounded corners are considered in Ref. [7, 8, 9]. A high order sandwich theory formulation for two composite facesheets separated by a core is used. The formulation takes into account the bending-stretching coupling of a facesheet in the case where a non-symmetric lay-up is used and also includes the transverse flexibility of the core material. Formulations are presented for the flat panels of the rectangular cross-section as well the rounded corners. The considered load case is an internal pressure. When assuming constant core and facesheet thicknesses along the fuselage perimeter, peak stresses are found in the longer side of the rectangle and important shear and transverse normal stresses are developed in the core of the curved corner regions. A BWB subjected to internal pressurisation is studied by finite element modelling (FEM) in Refs [2, 3, 4]. A fixed geometry with different constructions is considered: a double-skin ribbed shell versus a sandwich with honeycomb core, and a vaulted versus a flat design. An optimisation is performed via FEM to evaluate the weight of different fuselage concepts (circular, elliptical, multibubble). The results presented are mostly

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qualitative.

Elliptical pressurised vessels are also the subject of a few investigations. Expressions for the normal and shear forces as well as the bending moment along the perimeter of an elliptical pressurised vessel are developed in [10]. Strains are computed along the perimeter of an ellipse made of a symmetric, non-sandwich, composite material. The computed strains are then validated experimentally [11]. In a comparative study [12], the frames of an elliptical and a circular fuselage are sized for given fuselage width, internal pressure and floor loading. The considered construction is a standard monolithic stringer/frame reinforced skin. It is found that a significant weight penalty is incurred for the frames of the elliptical shape, especially if they are assumed to be made of aluminium.

Some authors use FEM to compute the stresses in ellipses made of an isotropic material for various values of eccentricity and thickness [13]. Ref [14] develops a superelement to efficiently compute the loads in a frame-stringer stiffened elliptical fuselage. The developed methodology is interesting but little calculations for a limited number of parameter values are shown. Patents were also obtained in relation with elliptical pressurised vessels. Ref. [15] proposes a non-circular fuselage composed of a sandwich structure in which the core thickness varies with the polar angle to follow the bending moment distribution. The bending moment generated by the internal pressure is known to vary with the polar angle, creating a positive (tensile) stress on one side of the shell and a negative (compressive) stress on the other side. At a certain polar angle, the bending moment is reversed and the sides of the shell subjected to tensile and compressive stresses are also reversed. The author proposes to use two materials for the facesheets, i.e., one material good at bearing tensile stresses and another one good at bearing compressive stresses. More recently, in Ref. [16], it is proposed to tailor structural attributes over the perimeter of a near elliptical fuselage in order to minimise the weight. However, no calculations or results are presented beyond the general concept.

What must be highlighted from this brief literature review is that although non-circular fuselages have often been considered, there is no published study which parametrically compare a non-circular fuselage to a circular one. There are many studies on *revolutionary* BWB aircraft designs, but what about an *evolutionary* change away from the conventional circular fuselage? Considering an existing circular fuselage design, if we incrementally move away from its circular shape, how much do the internal loads vary? How much structural weight must be added or subtracted to keep the maximum stresses constant? In other words, what's the marginal cost of ellipticity of a fuselage? In the present paper, we seek a simple demonstration of the structural benefit or cost in terms of weight of having an elliptical fuselage instead of a circular one.

We consider an elliptical fuselage cross-section made of a composite sandwich construction and develop a theoretical analysis to minimize its facesheet thickness in function of the polar angle, pressure loading, and ellipse eccentricity. A semi-analytical thin shell theory is applied [17] to compute the loading due to the pressurisation of the shell. A comparison of the proposed elliptical fuselage with a circular one for different scenarios is developed. We systematically analyse the effect of the ellipse eccentricity on the facesheet thickness and volume variations. The possibility of having an elliptical cross-section that would provide more space for the passengers while limiting the weight penalty compared to a circle is discussed.

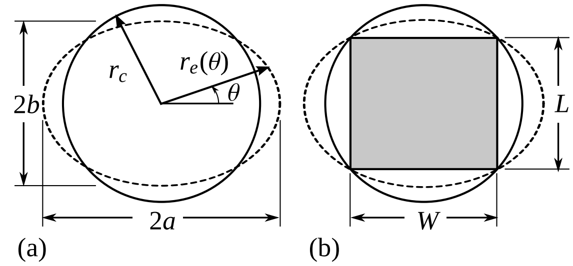


Figure 1: Schematics of the two comparison scenarios: (*a*) the circle and the ellipse have the *same area*; (*b*) the circle and the ellipse enclose the *same rectangle* of dimensions $W \times L$.

## 2. Methodology

We consider a non-reinforced thin shell fuselage of elliptical cross-section defined by its semi-major axis $a$, and semi-minor axis $b$. This ellipse traces the mean position through the thickness of the shell. It is compared with a circular cross-section of radius $r_c$. The radius of the elliptical cross-section with respect to the polar angle $\theta$ (see Fig. 1) is defined by

$$r_e = \frac{ab}{\sqrt{a^2 \sin^2\theta + b^2 \cos^2\theta}}. \tag{1}$$

The radius of curvature of an ellipse in polar coordinates is given by [18]

$$\bar{\rho} = \frac{1}{ab}\left(\frac{a^4 \sin^2\theta + b^4 \cos^2\theta}{a^2 \sin^2\theta + b^2 \cos^2\theta}\right)^{3/2}. \tag{2}$$

### 2.1. Comparison scenarios

The shell thickness is sized to carry only the load due to internal pressurisation of the fuselage. To compare the elliptical cross-section to a circular one, two comparison scenarios are defined (Fig. 1): same inner area, and same enclosed rectangle. The ellipse and circle of Fig. 1 (a) have the *same area*, i.e., $\pi ab = \pi r_c^2$. Thus, their large semi-major axis and radius are related by

$$\left.\frac{a}{r_c}\right|_{\substack{\text{same}\\ \text{area}}} = \sqrt{\frac{a}{b}}. \tag{3}$$

The ellipse and circle of Fig. 1 (b) have the *same enclosed rectangle* of dimensions $L$ by $W$. The smallest circle enclosing the rectangle obeys the relation

$$L^2 + W^2 = 4r_c^2. \tag{4}$$

An infinity of ellipses enclose the same rectangle, they obey the relation

$$(bW)^2 + (aL)^2 = 4\,(ab)^2\,. \tag{5}$$

By combining Eqs. 4 and 5, the ratio of the semi-major axis of the ellipse to the corresponding circle radius is expressed with respect to the rectangle aspect ratio $L/W$

$$\left.\frac{a}{r_c}\right|_{\substack{\text{same}\\\text{enclosed}\\\text{rectangle}}} = \sqrt{\frac{(W/L)^2 + (a/b)^2}{(W/L)^2 + 1}}. \tag{6}$$

### *2.2. Loads and construction*

Assuming a small shell thickness compared to the circle radius (or to the ellipse semi-minor axis), we can evaluate the loads due to an internal pressurisation $P$. An elliptical shell at equilibrium carries a tangential force per unit width $N$ varying with $\theta$ [10, 17]:

$$N = P\sqrt{a^2\cos^2\theta + b^2\sin^2\theta}, \tag{7}$$

The pressurisation gives rise to a distribution of bending moment per unit width. Upon integration of the equilibrium equations, the following distribution is obtained

$$M = \frac{Pa^2}{2}\left(D - \frac{P^2b^2}{N^2}\right), \tag{8}$$

where $D = 0.6592281\,(b/a - 0.26)^{2.14} + 0.653908$ is a fit of the integration constant obtained numerically for various values of the ratio $b/a$ [10]. At four points around the circumference of the ellipse, the bending moment changes sign. The angle where the bending moment is null depends only on the geometry:

$$\left.\cos^2(\theta)\right|_{M=0} = \left(\frac{1-D}{D}\right)\left(\frac{(b/a)^2}{1-(b/a)^2}\right). \tag{9}$$

To design the elliptical shell, three composite constructions are considered: monolithic, symmetric sandwich, and unsymmetric sandwich (Fig. 2). In the monolithic construction, the skin thickness of the ellipse $h_e$ varies with the polar angle $\theta$ and is sized to bear all circumferential loads. For both composite sandwich constructions, a core of uniform thickness $H$ is inserted between the facesheets. For the symmetric sandwich construction, the facesheet thickness is $h_e(\theta)/2$ on each side of the core. For the unsymmetric sandwich construction, the outer facesheet thickness is $h_o(\theta)$ and the inner facesheet thickness is $h_i(\theta)$. The total facesheet thickness in this case is $h_e(\theta) = h_o(\theta) + h_i(\theta)$.

To keep this analysis simple, we consider only the tangential stresses developing in the facesheets. We develop a curved beam stress formulation for an unsymmetric sandwich for which the symmetric and monolithic cases represent special cases. We assume that the facesheets behave according to the Euler-Bernoulli assumption, the core transverse stiffness is infinite, and the core has negligible in-plane stiffness [19]. The tangential stress in the elliptical shell at position $\theta$ about the circumference and at distance $\rho$ from the centre of curvature is due to the tangential force and the bending moment:

$$\sigma_e(\theta,\rho) = \frac{N}{h_e} - \frac{M\,(\rho - R)}{(h_i + h_o)\,(\bar{\rho} - R)\,\rho}, \tag{10}$$

where $R$ is the distance from the centre of curvature to the neutral surface, and where $\bar{\rho}$ is the distance from the centre of curvature to the centroid of the section. The radius of curvature $\bar{\rho}$ is defined in Eq. 2, and the radius $R$ can be evaluated with [19]

$$R = \frac{\int \mathrm{d}h}{\int \rho^{-1}\mathrm{d}h}, \tag{11}$$

where $\mathrm{d}h$ is an infinitesimal thickness of facesheet. Upon evaluating the integrals of Eq. 11 based on the dimensions given in Fig. 2, we obtain

$$R = \frac{h_i + h_o}{\ln\frac{\rho_j\rho_o}{\rho_i\rho_n}}, \tag{12}$$

where the radii drawn on Fig. 2 can be expressed as

$$\rho_i = \bar{\rho} - \frac{Hh_o}{h_i + h_o} - \frac{h_i + h_o}{2}, \tag{13}$$

$$\rho_j = \bar{\rho} - \frac{Hh_o}{h_i + h_o} + \frac{h_i - h_o}{2}, \tag{14}$$

$$\rho_n = \bar{\rho} + \frac{Hh_i}{h_i + h_o} + \frac{h_i - h_o}{2}, \tag{15}$$

$$\rho_o = \bar{\rho} + \frac{Hh_i}{h_i + h_o} + \frac{h_i + h_o}{2}. \tag{16}$$

Pressurisation always induces a positive tension $N$ in the shell. The maximum stress is reached at the innermost point in the inner facesheet $\rho = \rho_i$ for a positive bending moment or at the outermost point of the outer facesheet $\rho = \rho_o$ if the bending moment is negative.

For the sake of comparison, in a pressurised circular shell, the bending moment is neglected and the hoop stress is assumed constant through the thickness:

$$\sigma_c = \frac{Pr_c}{h_c}. \tag{17}$$

For all three constructions, the total volume of material in the facesheets can be evaluated as

$$V_e = 4\int_0^{\pi/2} r_e h_e \mathrm{d}\theta, \tag{18}$$

$$V_c = 2\pi r_c h_c, \tag{19}$$

for the ellipse and the circle, respectively. In our analysis, we use the total volume of facesheet as a proxy for the weight of the fuselage section. This is true if we assume a constant facesheet density and if we neglect the weight of the core in a sandwich. From a comparison standpoint, we consider that both the elliptical and the circular fuselages have the same core. Thus neglecting its weight in the comparison is reasonable.

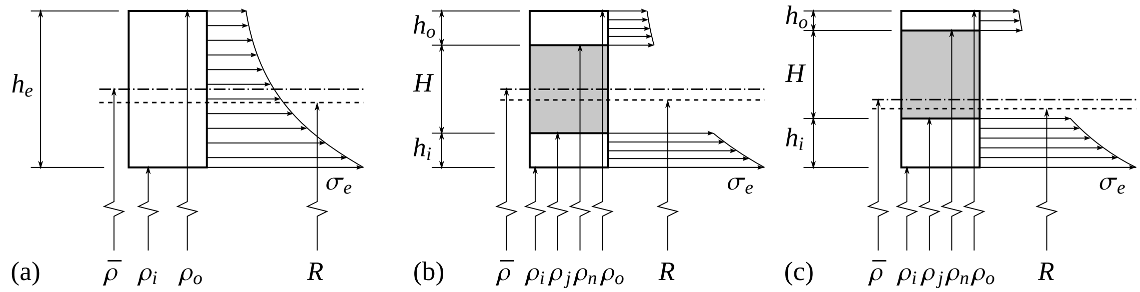


Figure 2: Schematic of the stress through the thickness for three elliptical shell constructions: (*a*) monolithic material; (*b*) symmetric sandwich; (*c*) unsymmetric sandwich. Distance from from the centre of curvature to the neutral surface (--) and to the centroid of the surface (-·-).

### *2.3. Dimensionless quantities*

To propose a systematic cost/benefit analysis in terms of weight for having an elliptical fuselage, proper dimensionless numbers are sought. We define the ellipse eccentricity as

$$\hat{e} = \sqrt{1 - (b/a)^2}. \tag{20}$$

We define the dimensionless internal loads as follows

$$\hat{N} = \frac{N}{Pa}, \quad \hat{M} = \frac{M}{Pa^2}. \tag{21}$$

To scale the loading we define the loading intensity

$$\hat{\Sigma} = \frac{\sigma_c}{P} = \frac{r_c}{h_c}. \tag{22}$$

The circular shell facesheet thickness $h_c$ is used to scale the dimensions of the elliptical fuselage

$$\hat{h}_e = \frac{h_e}{h_c}, \hat{h}_i = \frac{h_i}{h_c}, \hat{h}_o = \frac{h_o}{h_c}, \hat{H} = \frac{H}{h_c}, \hat{\rho} = \frac{\rho}{h_c}, \hat{R} = \frac{R}{h_c}. \tag{23}$$

The ratio of the ellipse semi-major axis to the circle radius varies according to the comparison scenario

$$\hat{a} = \frac{a}{r_c}, \quad \hat{r}_e = \frac{r_e}{r_c}. \tag{24}$$

By combining Eqs. 10 and 17, and by making use of the quantities of Eqs 20-24, the dimensionless stress in the facesheets of the ellipse can be written as

$$\hat{\sigma}_e = \frac{\sigma_e}{\sigma_c} = \frac{\hat{N}\hat{a}}{\hat{h}_i + \hat{h}_o} - \frac{\hat{M}\hat{\Sigma}\hat{a}^2\left(\hat{\rho} - \hat{R}\right)}{\left(\hat{h}_i + \hat{h}_o\right)\left(\hat{\bar{\rho}} - \hat{R}\right)\hat{\rho}}. \tag{25}$$

The relative volume of facesheet material required to design an elliptical shell with respect to a circular one is obtained by dividing Eq. 18 by Eq. 19

$$\frac{V_e}{V_c} = \frac{2}{\pi}\int_0^{\pi/2} \hat{r}_e\left(\hat{h}_i + \hat{h}_o\right) \mathrm{d}\theta. \tag{26}$$

### *2.4. Design criterion*

For all three constructions considered, we define a design criterion: we impose that for every polar angle $\theta$, the maximum stress of the elliptical shell is equal to the constant stress of the circular shell, i.e.,

$$\begin{cases} \hat{\sigma}_e(\theta,\hat{\rho})|_{\hat{\rho}=\hat{\rho}_i} = 1 & \text{if } \hat{M} > 0 \\ \hat{\sigma}_e(\theta,\hat{\rho})|_{\hat{\rho}=\hat{\rho}_o} = 1 & \text{if } \hat{M} < 0 \end{cases} \quad \forall\ \theta. \tag{27}$$

The criterion is always applied at the extreme fibre of the shell where the tension due to the moment $\hat{M}$ adds to the tension $\hat{N}$.

### *2.5. Numerical Solution*

For each of the two comparison scenarios (Eqs 3 and 6), and for the three construction cases, we seek to find the distribution along $\theta$ of the facesheet thickness. For the monolithic and symmetric sandwich constructions, we impose that $\hat{h}_i = \hat{h}_o = \hat{h}_e/2$. The core thickness $\hat{H}$ is null for the monolithic construction only. Then, we seek the facesheet thickness $\hat{h}_e$ which satisfies Eq. 27 within the acceptable tolerance. A Newton-Raphson algorithm is implemented in Matlab to iteratively converge to the right value of $\hat{h}_e$. This iterative loop is included in a continuation scheme in $\theta$. Starting from the angle where the bending moment is null (Eq. 9), the value of $\theta$ is incremented and the solution $\hat{h}_e$ from the previous angle is used as a guess for the next one.

For the unsymmetric sandwich construction, solution is more complicated to obtain because both facesheet thicknesses $\hat{h}_i$ and $\hat{h}_o$ must be solved for. So, for a positive bending moment $\hat{M} > 0$, a value for the outer facesheet thickness $\hat{h}_o$ is guessed and the Newton-Raphson procedure is used to find the corresponding $\hat{h}_i$ value which insures that the criterion in Eq. 27 is met. This procedure is then included in a loop using the *fminsearch* function of Matlab to vary the thickness $\hat{h}_o$ to find the value which minimises the total cost function

$$\mathcal{F} = \hat{h}_i + \hat{h}_o + W_1\,|\hat{\sigma}_e + 1| \times \begin{cases} 1 & \text{if } |\hat{\sigma}_e|_{\hat{\rho}=\hat{\rho}_o} < -1 \\ 0 & \text{if } |\hat{\sigma}_e|_{\hat{\rho}=\hat{\rho}_o} > -1 \end{cases}, \tag{28}$$

where $W_1$ is a constant penalty weight. In the cost function, the first two terms on the RHS insure that the total thickness of facesheets is minimised, whereas the third term enforces that $\hat{h}_o$ is sufficiently thick to avoid the compressive stress at $\hat{\rho} = \hat{\rho}_o$ to

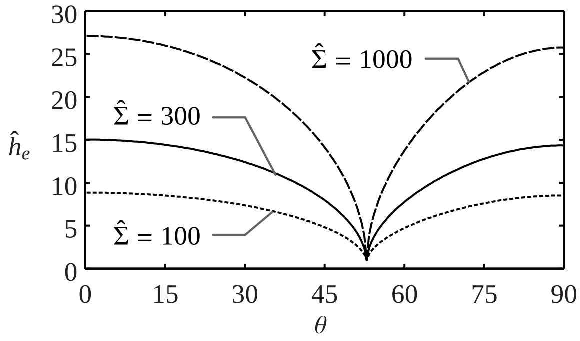


Figure 3: Polar distribution of the thickness ratio $\hat{h}_e$ between the elliptical and circular fuselages for the same area scenario, monolithic construction, eccentricity $\hat{e} = 0.6$, and varying loading intensity $\hat{\Sigma}$.

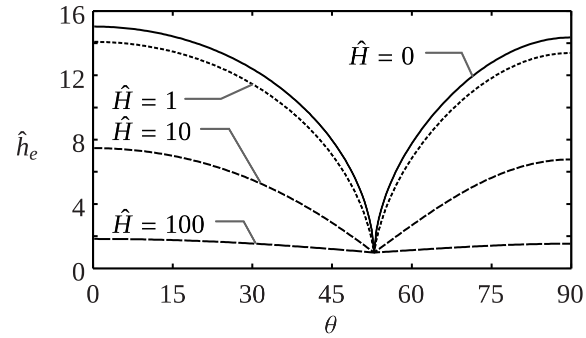


Figure 4: Polar distribution of the thickness ratio $\hat{h}_e$ between the elliptical and circular fuselages for the same area scenario, symmetric sandwich construction, eccentricity $\hat{e} = 0.6$, loading intensity $\hat{\Sigma} = 300$, and varying core thickness $\hat{H}$.

become larger in magnitude than the uniform tensile stress in the benchmark circular shell. The constant $W_1$ is set arbitrarily at a large enough value of $W_1 = 100$ which insures that the function $\mathcal{F}$ will only be minimised if $|\hat{\sigma}_e|_{\hat{\rho}=\hat{\rho}_o} > -1$. Changing the value of $W_1$ to 200 does not influence the final result appreciably. The nested Newton-Raphson and minimisation loops are included in a continuation procedure for $\theta$ starting from the angle where the bending moment is null (Eq. 9) similarly as for the previous constructions. For a negative bending moment $\hat{M} < 0$, the thickness values $\hat{h}_i$ and $\hat{h}_o$, and the extreme fibres $\hat{\rho}_i$ and $\hat{\rho}_o$ are inverted in the algorithm.

Once the thickness distribution along $\theta$ for the ellipse is found, Eq. 26 is integrated numerically to find the volume ratio.

## 3. Results

We present comparisons between the elliptical and circular shell thicknesses for the three construction cases and two scenarios developed. Various values of eccentricity, core thickness and loading intensity are selected and a case study with typical business jet dimensions is discussed.

### *3.1. Monolithic construction*

We first analyse the results of the same area scenario. For the monolithic construction, the variation with $\theta$ of the elliptical shell thickness in comparison with the constant circular shell thickness is shown in Fig. 3. The results are presented for three different loading intensities. The relative thickness $\hat{h}_e$ increases with the loading intensity $\hat{\Sigma}$. With a large loading intensity ($\hat{\Sigma} = 1000$), the thickness of the ellipse varies between 1 and 27 times that of the circle. For a small loading intensity ($\hat{\Sigma} = 100$), the thickness of the ellipse varies between 1 and 9 times that of the circle. The polar angle where the ellipse skin thickness is minimum and equal to that of the circle (Eq. 9) is $\theta_{M=0} \simeq 53°$ for the eccentricity selected here ($\hat{e} = 0.6$). Even for small loading intensity ($\hat{\Sigma} = 100$), the thickness ratio is well above 1, i.e., the thickness of the elliptical shell is much greater than that of the circular one. Because of the inefficiency of monolithic thin shells to carry bending moments, we do not consider monolithic constructions any further.

### *3.2. Symmetric sandwich construction*

We now analyse a symmetric sandwich construction where the outer and inner facesheets have the same thickness and are separated by a core. We assume that the stress due to the pressure is bore by the facesheets only and that the core has negligible weight compared to the facesheets. The variation with $\theta$ of the relative thickness of the elliptical shell with respect to the circular shell is shown in Fig. 4. Note that the scale of the ordinate in Fig. 4 is different from that in Fig. 3. The results of Fig. 4 are presented for four different core thicknesses $\hat{H}$, and for the same area scenario. Without a core ($\hat{H} = 0$), the curve is identical to that of monolithic construction of Fig. 3 as expected. For a thin core ($\hat{H} = 1$), the elliptical shell is up to 14 times the thickness of the circular shell. With a thick core ($\hat{H} = 100$), the elliptical shell has a thickness varying between 0.98 and 1.9 that of the circle. Therefore, even for the thick core, the skin of the elliptical shape is mostly thicker than that of the circular shell. Again, all four curves have a discontinuity and a minimum at $\theta \simeq 53°$.

As the maximum stress equivalent to the circle hoop stress is found at the outer *or* inner edge of the sandwich, the opposing facesheet is underused, therefore the symmetric sandwich construction is not optimal. So we turn our attention to an unsymmetric sandwich construction.

### *3.3. Unsymmetric sandwich construction*

The facesheet thicknesses of the unsymmetric sandwich construction with the same area scenario are shown on Fig. 5 for an eccentricity $\hat{e} = 0.6$, loading intensity $\hat{\Sigma} = 300$ and core thickness $\hat{H} = 15$. The total facesheet thickness $\hat{h}_e$ is obtained by summing the inner and outer facesheet thicknesses ($\hat{h}_i + \hat{h}_o$). For $\theta$ varying between 0 and 53°, the outer facesheet is thicker than the inner facesheet as the bending moment is negative and gives rise to tensile loads in the outer facesheet. For $53° < \theta \leq 90°$, the bending moment is positive and the inner facesheet is thicker than the outer facesheet. Comparison with the symmetric sandwich construction reveals a weight saving of the order of 11% for the selected set of parameters. But, the relative thickness $\hat{h}_e$ is mostly larger than 1.0, varying between 0.98 and 5.2.

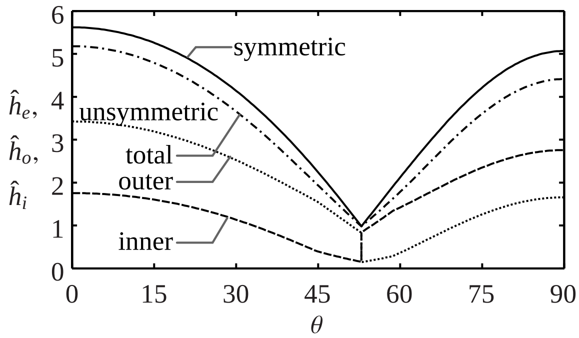


Figure 5: Polar distribution of the thickness ratio $\hat{h_e}$ between the elliptical and circular fuselages for the symmetric and unsymmetric sandwich constructions for the same area design scenario, with eccentricity $\hat{e} = 0.6$, loading intensity $\hat{\Sigma} = 300$, and core thickness $\hat{H} = 15$.

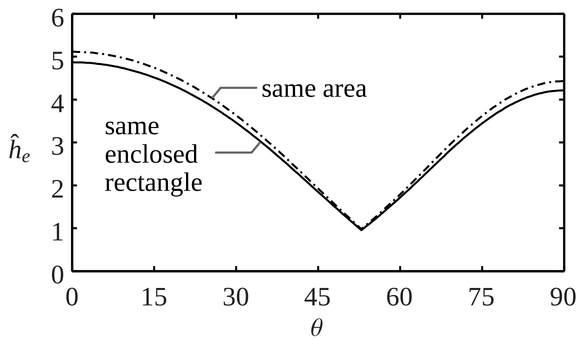


Figure 6: Polar distribution of the thickness ratio $\hat{h_e}$ between the elliptical and circular fuselages for the same area and same enclosed rectangle design scenarios with unsymmetric sandwich construction, eccentricity $\hat{e} = 0.6$, loading intensity $\hat{\Sigma} = 300$, core thickness $\hat{H} = 15$ and a rectangle of aspect ratio $L/W = 0.7$.

The unsymmetric sandwich construction with the same enclosed rectangle scenario is considered (Fig. 6). A rectangle aspect ratio $L/W = 0.7$ is selected and comparison with the same inner area scenario is shown. The same enclosed rectangle scenario presents some benefits compared to the same inner area with a relative thickness varying between 0.96 and 4.9, as compared to 0.98 and 5.1 for the same area. Once again, the relative thickness is mostly larger than 1.0, which represents a weight penalty compared to the circular shell.

To better evaluate this penalty, the volume of the facesheets of the ellipse normalised by that of the circle is evaluated using Eq. 26. This normalised volume is shown in Fig. 7 as a contour plot in function of the core thickness and the ellipse eccentricity for the same enclosed rectangle scenario. Note that for the same area scenario (not shown), the graph is similar but the isolines are slightly shifted left. In Fig. 7, for the range of parameters tested, the volume of the skin of the elliptical shape is always larger than that of the circular shape. However, for small values of eccentricity, there is a contour line of $V_e/V_c = 1.01$ beyond which the elliptical shape is less than 1% heavier than the circular shape. If the elliptical cross section design brings other benefits, a small weight gain could be acceptable.

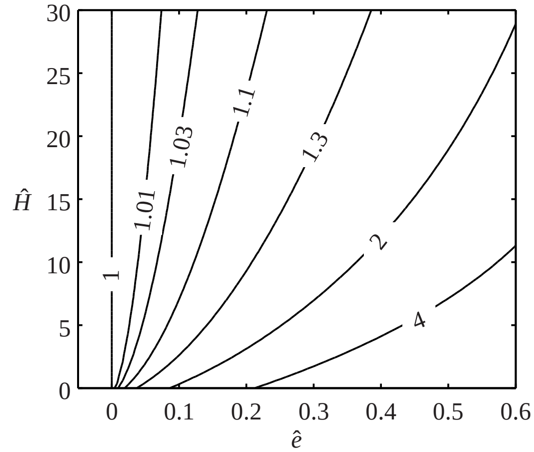


Figure 7: Isolines of the elliptical shell facesheet volume normalised by the circular shell volume ($V_e/V_c$) for the same enclosed rectangle scenario and unsymmetric sandwich construction, with loading intensity $\hat{\Sigma} = 300$ and rectangle aspect ratio $L/W = 0.7$.

### 3.4. *Case study on a typical business jet*

We consider a typical circular fuselage of a business jet with a diameter of 2 m. We assume a sandwich construction with a total skin thickness of $h_c = 3$ mm. So the loading intensity $\hat{\Sigma}$ is calculated as $r/h_c = 300$.

A rectangle of width $W = 1.7$ m and height $L = 1.2$ m is inserted in the circular fuselage (rectangle aspect ratio is $L/W = 0.7$). We replace the circular shell for an elliptical one of unsymmetric sandwich construction.

We look at Fig. 7 for a rectangle $L/W = 0.7$ and $\hat{\Sigma} = 300$. An elliptical shape with a core of $\hat{H} = 10$ and an eccentricity of $\hat{e} = 0.4$, which corresponds to a ratio $b/a = 0.92$, will inflict a weight penalty of around 70% as compared to a circular shape. If we consider a thicker core $\hat{H} = 15$ and a smaller eccentricity of $\hat{e} = 0.15$, then we can read off of Fig. 7 that the volume ratio is 1.1, i.e., a weight penalty of 10% as compared to a circle.

## 4. Conclusion

We considered a pressurised aircraft fuselage and analysed the benefits or costs of moving away from a circular to an elliptical cross-section. The considered load case was an internal pressure and we looked at the cross-section of the fuselage only meaning that longitudinal loads were not accounted for. The structure was assumed to be made of a composite material in the form of a monolithic or sandwich construction in which two facesheets are separated by a lightweight core. A curved beam stress formulation was developed for an unsymmetric sandwich construction in which the outer and inner facesheets have different thicknesses. The monolithic and symmetric sandwich constructions (i.e. a sandwich with the same facesheets thickness on both sides of the core) represented special cases of the formulation. The elliptical shell made of a monolithic construction proved disadvantageous compared to a circle as the elliptical shell needs to be much thicker than the circular shell to bear the same internal pressure. The symmetric sandwich construction reduced the weight penalty compared to the monolithic construction due to the high bending stiffness the core provides. Nevertheless, an important increase of the facesheets thickness was required compared to the circular shape. In a third analysis, we finally considered an unsymmetric sandwich construction. Weight penalties as compared to a circular shell were still obtained, but under certain sets of parameters (low eccentricity, thick core, low loading intensity), the weight gain can be as small as 1% compared to a circle. If the ellipse brings other benefits like increased seating and optimised use of interior space, then a small weight penalty could be acceptable.

Important simplifications were done to keep the argument simple. Firstly, only circumferential loads due to pressurisation were considered. In addition to circumferential stress, pressurisation will give rise to longitudinal stress along the axis of the fuselage. In the same area design scenario, this longitudinal stress should be similar for both the circular and the elliptical cross-section. But, a proper stacking order (putting the plies aligned with the fuselage axis next to the sandwich core) could be used to increase the distance of the circumferential plies from the neutral axis and render them more effective at bearing bending moments. Moreover, in the same enclosed rectangle scenario, because the cross-sectional area of the ellipse can be smaller than that of the circle, the longitudinal stress can be expected to be lower for the ellipse and require less plies. Secondly, a fuselage is sized based on internal pressure but also on longitudinal bending moment and torsion. These load cases should be considered in the future to better assess the advantages or disadvantages of an elliptical cross-section. Furthermore, in a composite sandwich structure, the outer facesheet is sized based on potential debris impacts. These considerations lead to thicker facesheets of the circular fuselage than what was assumed here. As a more complicated stress state is considered, there is more gain to be made from an optimised stacking sequence. So the weight penalty of the elliptical fuselage computed here should be regarded as an upper limit and could be reduced or eliminated when all other design factors are taken into account.

Modern multidisciplinary design optimisation makes extensive use of computational models including the finite element method to optimise structures in such a way that they meet a number of metrics [20]. In order to speed up the optimisation scheme, we have developed a simple methodology that can be easily implemented to define the boundaries of the geometrical parameters to be optimised. The presented results are general and do not focus on any particular fuselage dimension; therefore they are valid for a wide range of cases.

Future work should explore the ply stacking sequence of the elliptical sandwich when the longitudinal stresses arising both from pressurisation and from longitudinal bending are considered. The core in the sandwich which must bear the brunt of the shear force [10, 17] should be sized and its weight evaluated.

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